PROJECTED WRITTEN NOTES FROM THE M 408D LECTURE ON THURSDAY, JANUARY 18, 2024, ON MORE ON INTEGRATION BY PARTS, ON INTEGRATING PRODUCTS AND POWERS OF TRIG FUNCTIONS, AND SIMLIFYING TRIGFUNC(INVERSETRIG(x)). CLASS #2

MORE ON "Integration by Parts"

EXAMPLE 2 from "Two Elaborate "Parts" Examples"

$$\int e^x \cos x\,dx = e^x \sin x - \int e^x \sin x\,dx$$

$$\left[\begin{array}{ll} u = e^x & dv = \cos x\,dx \\ du = e^x dx & v = \sin x \end{array}\right]$$

$$\left[\begin{array}{ll} u = e^x & dv = \sin x\,dx \\ du = e^x dx & v = -\cos x \end{array}\right]$$

$$\int e^x \cos x\,dx = e^x \sin x - \left[-e^x \cos x - \int -e^x \cos x\,dx\right]$$

$$\int e^x \cos x\,dx = e^x \sin x + e^x \cos x - \int e^x \cos x\,dx$$

add to both sides

$$2\int e^x \cos x\,dx = e^x \sin x + e^x \cos x + C$$

$$\int e^x \cos x\,dx = \frac{1}{2} e^x \sin x + \frac{1}{2} e^x \cos x + \cancel{\tfrac{1}{2}}C$$

$$\int e^x \cos x\,dx = \frac{1}{2} e^x (\sin x + \cos x) + C$$

Sec 7.2 Integrating Products and Powers of Trig Functions

See the handout in the "Techniques of Integration" Folder, "Integrating Products and Powers of Trig Functions".

Reviewing Trig Identities

(1) Recall $\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$

$\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$

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(2) $\sec^2 x$ vs. $\tan^2 x$?

$\frac{1}{\cos^2 x}\left[\cos^2 x + \sin^2 x = 1\right] = \left[1 + \tan^2 x = \sec^2 x\right]$

$1 + \tan^2 x = \sec^2 x$ and $\tan^2 x = \sec^2 - 1$

We will need these integrals:

$\int \tan x\,dx = \ln|\sec x| + C$

$\int \sec x\,dx = \ln|\sec x + \tan x| + C$

(See p 3 of "The Handout")

Methods: (m, n, k, t are positive integer exponents)

For Type I: $\int (\sin^m x)(\cos^n x)\,dx$ (p.1)

For Type II: $\int (\tan^m x)(\sec^n x)\,dx$ (p2)

For Type III: $\int (\text{Trig function})^k\,dx$ (p3)

Type IA EXAMPLE: odd is NICE!

$$\int \sin^8 x \cos^3 x\,dx = \int \sin^8 x (\cos^2 x)\underbrace{(\cos x)\,dx}_{du}$$

$u = \sin x$

$du = \cos x\,dx$

Write all $\cos^2 x$ factors as $(1-\sin^2 x)$

$$= \int \sin^8 x\,(1-\sin^2 x)(\cos x)\,dx$$

$$= \int u^8(1-u^2)\,du$$

$$= \int (u^8 - u^{10})\,du$$

$$= \frac{1}{9}u^9 - \frac{1}{11}u^{11} + C$$

$$= \frac{1}{9}\sin^9 x - \frac{1}{11}\sin^{11} x + C$$

A Type IIA Example:

EVEN is NICE!

$$\int \tan^3 x \sec^4 x\,dx$$

$$= \int \tan^3 x\,(\sec^2 x)\underbrace{(\sec^2 x)\,dx}_{du}$$

u = tan x
du = sec²x dx
Write each other sec²x factor as (1 + tan²x)

$$= \int \tan^3 x\,(1+\tan^2 x)(\sec^2 x)\,dx$$

$$= \int u^3(1+u^2)\,du$$

$$= \int (u^3+u^5)\,du$$

$$= \frac{1}{4}u^4 + \frac{1}{6}u^6 + C$$

$$= \frac{1}{4}(\tan^4 x) + \frac{1}{6}\tan^6 x + C$$

Type III C Example:

$$\int \tan^3 x\,dx = \int (\tan^2 x)(\tan x)\,dx$$

$$= \int (\sec^2 x - 1)\tan x\,dx$$

$$= \int \tan x\,\underbrace{(\sec^2 x)\,dx} - \int \tan x\,dx$$

$u = \tan x$

$du = \sec^2 x\,dx$

$$= \int u\,du - (\ln|\sec x|)$$

$$= \frac{1}{2}u^2 - \ln|\sec x| + C$$

$$\int \tan^3 x\,dx = \frac{1}{2}(\tan^2 x) - \ln|\sec x| + C$$

"DRAWING THE TRIANGLE TECHNIQUE"

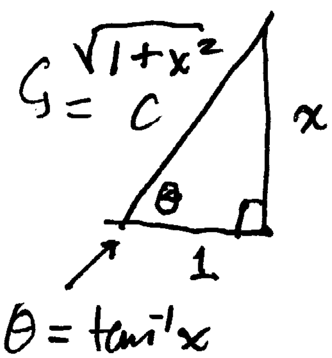
An expression like $\cos(\tan^{-1} x)$

can be simplified to an algebraic expression

How? Simplify $\cos(\tan^{-1} x)$.

Write $\theta = \tan^{-1} x$. Then, by definition of $\tan^{-1} x$

$$\tan\theta = x = \frac{x}{1} =$$



By the Pythagorean Theorem,

$$c^2 = 1^2 + x^2$$

$$c = \sqrt{1+x^2}$$

$$\cos(\tan^{-1} x) = \cos\theta = \frac{\text{Adj}}{\text{Hyp}} = \frac{1}{\sqrt{1+x^2}}$$

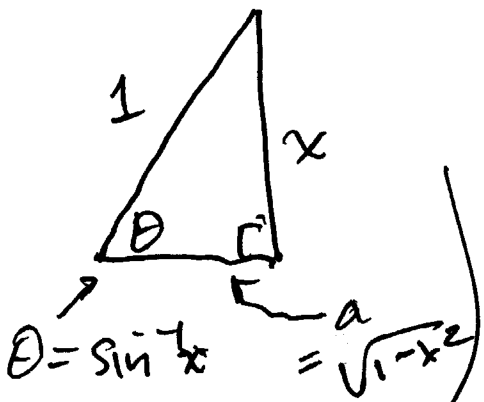
$$\boxed{\cos(\tan^{-1} x) = \frac{1}{\sqrt{1+x^2}}}$$

Simplify: $\sec(\sin^{-1}x)$:

Sol'n

Write $\theta = \sin^{-1}x$

So, $\sin\theta = x = \frac{x}{1} = \frac{\text{Opp}}{\text{Hyp}}$



$1^2 = a^2 + x^2$

$a^2 = 1 - x^2$

$a = \sqrt{1-x^2}$

$\sec(\sin^{-1}x) = \sec\theta = \frac{\text{Hyp}}{\text{Adj}} = \frac{1}{\sqrt{1-x^2}}$

$$\boxed{\sec(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}}$$